

Pearson Edexcel International GCSE

| Morning (Time: 2 hours) | Paper Reference 4PM0/01 |
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## Further Pure Mathematics

Paper 1

Calculators may be used.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
- there may be more space than you need.


## Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.



## Answer all TEN questions.

## Write your answers in the spaces provided.

You must write down all the stages in your working.


Diagram NOT accurately drawn

Figure 1
Figure 1 shows a sector $O A B$ of a circle, centre $O$.
The area of the sector is $27 \mathrm{~cm}^{2}$
The size of angle $A O B$ is 1.5 radians.
Find the perimeter of the sector.

Question 1 continued

2 The sum of the first $n$ terms of an arithmetic series is $S_{n}$
Given that $S_{n}=\sum_{r=1}^{n}(4 r+1)$
(a) show that $S_{n}=n(3+2 n)$

The $r$ th term of this arithmetic series is $t_{r}$
Given that $S_{n+3}=S_{n}+3 t_{15}$
(b) find the value of $n$.

Question 2 continued

Question 2 continued

Question 2 continued

$$
\mathrm{f}(x)=(2 x+1)\left(x^{2}+5 x-3\right)
$$

(a) Show that $\mathrm{f}(x)=2 x^{3}+11 x^{2}-x-3$
(b) Hence use algebra to solve the equation $2 x^{3}+11 x^{2}-x-3=0$

Give your roots to 3 decimal places where appropriate.

Question 3 continued

$$
\begin{aligned}
\sin (A+B) & =\sin A \cos B+\sin B \cos A \\
\tan A & =\frac{\sin A}{\cos A}
\end{aligned}
$$

(a) Show that the equation $a \sin (x-30)^{\circ}=b \sin (x+30)^{\circ}$
can be written in the form $\quad \tan x^{\circ}=\frac{a+b}{\sqrt{3}(a-b)}$


Figure 2

In triangle $A B C, A C=6 \mathrm{~cm}, B C=14 \mathrm{~cm}, \angle A B C=(x-30)^{\circ}$ and $\angle B A C=(x+30)^{\circ}$ as shown in Figure 2.
(b) Find, in degrees to 1 decimal place, the size of $\angle A C B$.
(c) Find, to 3 significant figures, the area of triangle $A B C$.

Question 4 continued

Question 4 continued

Question 4 continued

$$
\mathrm{f}(x)=2 x^{2}+7 x-4
$$

Given that $\mathrm{f}(x)$ can be written in the form $A(x+B)^{2}+C$
(a) find the value of $A$, the value of $B$ and the value of $C$.
(b) Write down
(i) the minimum value of $\mathrm{f}(x)$,
(ii) the value of $x$ at which this minimum occurs.

The equation $\mathrm{f}(x)=p x-6$ has unequal real roots.
(c) Find the set of possible values of $p$.

Question 5 continued

Question 5 continued

Question 5 continued

6 Given that $y=x^{2} \sqrt{(2 x-3)}$
(a) show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{x(5 x-6)}{\sqrt{(2 x-3)}}$
(b) find the value of $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $x=2$

The curve $C$ has equation $y=x^{2} \sqrt{(2 x-3)}$
(c) Find an equation of the normal to $C$ at the point on $C$ where $x=2$

Give your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.

Question 6 continued

Question 6 continued

Question 6 continued


## Figure 3

Figure 3 shows a rectangular sheet of metal 10 cm by 16 cm . A square of side $x \mathrm{~cm}$ is cut away from each corner of the sheet. The sheet is then folded along the dotted lines to form an open box.

The volume of the box is $V \mathrm{~cm}^{3}$
(a) Show that $V=4 x^{3}-52 x^{2}+160 x$
(b) Using calculus, find the value of $x$ for which $V$ is a maximum, justifying that this value of $x$ gives a maximum value of $V$.
(c) Find the maximum value of $V$.

Question 7 continued

Question 7 continued

Question 7 continued

8 A curve $C$ has equation $y=\frac{5 x-3}{2 x-1} \quad x \neq \frac{1}{2}$
(a) Write down an equation of the asymptote to $C$ that is
(i) parallel to the $y$-axis,
(ii) parallel to the $x$-axis.
(b) Find the coordinates of the points of intersection of $C$ with the coordinate axes.
(c) Using calculus show that at every point on the curve, the gradient of $C$ is positive.
(d) Using the axes on the opposite page, sketch $C$, showing clearly the asymptotes and the coordinates of the points of intersection of $C$ with the coordinate axes.

The line $l$ is the tangent to $C$ at the point on the curve where $x=1$
(e) Find an equation of $l$, giving your answer in the form $y=m x+c$

Question 8 continued

Question 8 continued

Question 8 continued

9 The point $A$ has coordinates $(-3,-6)$ and the point $B$ has coordinates $(5,-2)$
The line $l$ passes through the point $A$ and the point $B$.
(a) Find an equation of $l$, giving your answer in the form $y=m x+c$

The point $P$ has coordinates $(k,-2)$. The line through $A$ and $P$ is perpendicular to $l$.
(b) Show that $k=-5$

The point $Q$ has coordinates $(e, f)$. The line through $B$ and $Q$ is also perpendicular to $l$.
Given that the length of $P Q$ is $\sqrt{85}$ and that $f>0$
(c) find the coordinates of $Q$.
(d) Calculate the area of quadrilateral $A B Q P$.

Question 9 continued

Question 9 continued

Question 9 continued

10 (a) Expand $(1-2 x)^{-\frac{1}{2}}$ in ascending powers of $x$ up to and including the term in $x^{3}$, simplifying each term as far as possible.
(b) Write down the range of values of $x$ for which your expansion is valid.

$$
f(x)=\frac{2-x^{2}}{\sqrt{(1-2 x)}}
$$

(c) Find the series expansion of $\mathrm{f}(x)$ in ascending powers of $x$ up to and including the term in $x^{3}$, simplifying each term as far as possible.

The region $R$ is bounded by the curve with equation $y=\mathrm{f}(x)$, the positive $x$-axis, the positive $y$-axis and the line with equation $x=0.2$
(d) Using your expansion of $\mathrm{f}(x)$ and algebraic integration, find an estimate for the area of $R$, giving your answer to 4 decimal places.

Question 10 continued

Question 10 continued

